**1.** (10 marks) The 6-month and 1-year zero rates are both **12%** per annum. For a bond that has a life of **18** months and pays a coupon of **8%** per annum (with semiannual payments and one having just been made), the yield is **10.6%** per annum. What is the bond’s price? What is the **18-month** zero rate? All rates are quoted with semiannual compounding.

Suppose we assume the bond a has face value of $100.   
We will get $4 every 6 months (8% per annum coupon rate).  
Yield is 10.6% per **annum**.

**NPV** of this bond will be  
6 months = 4 / (1 + (0.106 / 2)) +   
12 months = 4 / (1 + (0.106 / 2)) ^ 2 +   
18 months = (4 + 100) / (1 + (0.106 / 2)) ^ 3  
**Bond’s Price** = **$96.4795**

If **18-month** zero rate is **R**,  
6-month and 1-year zero rates are both **12%**

**NPV will be**6 months = 4 / (1 + (0.12 / 2)) +   
12 months = 4 / (1 + (0.12 / 2)) ^ 2 +   
18 months = (4 + 100) / (1 + (**R** / 2)) ^ 3  
 = 96.4795  
R will be = 0.10542

**18-month zero rate** = **10.542%**

2. (10 marks) A stock price is currently **$30**. It is known that at the end of two months it will be either **$27** or **$34**. The risk-free interest rate is **12%** per annum with continuous compounding. Suppose ST is the stock price at the end of two months. What is the value of a derivative that pays off ST2 at this time?

Current price = $30

Up price = $34

Down price = $27

Risk-free rate = 12%

Option price = f

T = 2 months

For a riskless portfolio = -1 option

34 Δ – 6 = 27 Δ

Δ = 0.85

Present Value of portfolio today is

= (27 \* 0. 85) e ^ (-0.12 \* 2 / 12)

= 22.95 e ^ (-0.12 \* 2 / 12)

= 22.495

With current stock price, the value of Portfolio today is also

= 30 \* 0.85 – f

So, 22.495 = 30 \* 0.85 – f

f = 3.005

The derivative that pays off ST2

= **3.005**

3. (10 marks) Draw a diagram showing the variation of an investor’s profit and loss with the terminal stock price for a portfolio consisting of

(a) One share and a short position in one call option  
(b) Three shares and a short position in two call options   
(c) Two shares and a short position in three call options   
(d) Two shares and a short position in four call options

A picture containing text, map, different, various

Description automatically generated

4. (10 marks) A stock price is currently **$50**. It is known that at the end of **one** month it will be either **$52.5** or **$47.5**. The risk-free interest rate is **9%** per annum with continuous compounding. Use the no-arbitrage approach to find the value of a one-month European call option with a strike price of **$49**?

Current price = $50

Up price = $52.5

Down price = $47.5

Risk-free rate = 9%

Strike price = $49

Option price = f

T = 1 months

For a riskless portfolio,= -1 option

52.5 Δ – 4 = 47.5 Δ

Δ = 0.8

Present Value of portfolio today is

= (47.5 \* 0. 8) e ^ (-0.09 \* 1 / 12)

= 38 e ^ (-0.0075)

= 37.716

With current stock price, the value of Portfolio today is also

= 50 \* 0.8 – f

So, 37.716 = 50 \* 0.8 – f

f = 2.283

The value of a one-month European call option is

= **2.283**

5. (10 marks) A stock price is currently **$60**. It is known that over each of the next **two three-month periods** it will go up by **10%** or down **8%**. The risk-free interest rate is **11%** per annum with continuous compounding. Use the **risk-neutral** approach to find the value of a **six-month European** put option with a strike price of **$60**?

u = 0.1 (10%)

d = 0.08 (8%)

S = 60

T = 0.25 (3 months periods)

R = 0.11 (11%)

S \* e ^ (R \* T) = Su \* p + (Sd \* (1 - p))

60e^(0.11 \* 0.25) = (66 \* p) + (55.2 \* (1- p))

61.672 = 66p + 55.2 – 55.2p

6.472 = 10.8p

p = 0.5993

A picture containing chart

Description automatically generated

**Put Option** = e ^ -2 (0.11 \* 0.25) \* ((0.5993 ^ 2 \* 0) + (2 \* (0.5993 \* (1 – 0.5993)) \* 1) + ((1 – 0.5993) ^ 2) \* 9.216)

= 0.9464 \* (0.48072 + 1.4797)

= **1.8553**

**6.** (10 marks) A bank loan department is trying to determine the correct rate for a 2-yr loan to be made two years from now. If current zero rates are 1-yr = 2%, 2-yr = 3%, 3-yr = 3.5%, 4-yr = 4.5%, the foward rate for the loan should be?

|  |  |
| --- | --- |
| Years | Zero rates |
| 1 | 2% |
| 2 | 3% |
| 3 | 3.5% |
| 4 | 4.5% |

Rf2 = (R2 \* T2 – R1 \* T1) / (T2 – T1)

= (0.03 \* 2 – 0.02 \* 1) / (2 - 1)

= 0.06 – 0.02 / 1

= 0.04

= **4%**

**7.** (10 marks) Suppose that the risk-free interest rate is **9%** per **annum** with continuous compounding and that the dividend yield on a stock index is **3%** per annum. The index is standing at **500**, and the futures price for a contract deliverable in **four** months is **520**. What arbitrage opportunities does this create?

Suppose FV = 500 \* e ^ ((0.09 – 0.03) \* 4 / 12)

= 500 \* e ^ 0.02

= **510.099**

Since Future price for a contract = **520**

The index’s future value is lower than contract’s future price (510.099 < 520)

The correct arbitrage should be

1. **Sell futures** contract
2. **Buy the stocks** underlying the index

**8.** The **6-month, 12-month, 18-month,** and **24-month** zero rates are **5%**, **5.5%**, **5.75%,** and **6%**, with semiannual compounding.

(a) What are the **rates** with continuous compounding?   
(b) What is the forward rate for the **6-month** period beginning in **18** months?   
(c) What is the value of an FRA that promises to pay you **6.5%** (compounded semiannually) on a principal of **$1** million for the **6-month** period starting in **18 months**?

(a) **Continuous compounding** = 2ln(1 + R / 2)

**6-month** = 2ln(1 + (0.05 / 2))

= 0.04938

= **4.938%**

**12-month** = 2ln(1 + (0.055 / 2))

= 0.05425

= **5.425%**

**18-month** = 2ln(1 + (0.0575 / 2))

= 0.05668

= **5.668%**

**24-month** = 2ln(1 + (0.06 / 2))

= 0.05911

= **5.911 %**

|  |  |
| --- | --- |
| Months | Zero rates |
| 6 | 5% |
| 12 | 5.5% |
| 18 | 5.75% |
| 24 | 6% |

(b)

Forward rate = (R24 \* T24 – R18 \* T18) / (T24 – T18)

= (0.05911 \* 2 – 0.05668 \* 1.5) / 2 – 1.5

= 0.0664

= **6.64% (Continuous compounding)**

For semi annual

0.0664 = 2ln(1 + R/2)

0.0332 = ln(1 + R/2)

e ^ 0.0332 = 1 + R/2

R = 0.0675

= **6.75% (Semiannual compounding)**

(c) Since Forward rate for 18-months is 6.75% but FRA only give 6.5%, we need to pay difference of interest

= 1,000,000 \* (0.065 – 0.0675) \* 6/12 \* (e ^ - 0.05911 \* 2)

= -$1,110.6393