**1.** (10 marks) The 6-month and 1-year zero rates are both **12%** per annum. For a bond that has a life of **18** months and pays a coupon of **8%** per annum (with semiannual payments and one having just been made), the yield is **10.6%** per annum. What is the bond’s price? What is the **18-month** zero rate? All rates are quoted with semiannual compounding.

Suppose we assume the bond a has face value of $100.   
We will get $4 every 6 months (8% per annum coupon rate).  
Yield is 10.6% per **annum**.

**NPV** of this bond will be  
6 months = 4 / (1 + (0.106 / 2)) +   
12 months = 4 / (1 + (0.106 / 2)) ^ 2 +   
18 months = (4 + 100) / (1 + (0.106 / 2)) ^ 3  
**Bond’s Price** = **$96.4795**

If **18-month** zero rate is **R**,  
6-month and 1-year zero rates are both **12%**

**NPV will be**6 months = 4 / (1 + (0.12 / 2)) +   
12 months = 4 / (1 + (0.12 / 2)) ^ 2 +   
18 months = (4 + 100) / (1 + (**R** / 2)) ^ 3  
 = 96.4795  
R will be = 0.10542

**18-month zero rate** = **10.542%**

2. (10 marks) A stock price is currently **$30**. It is known that at the end of two months it will be either **$27** or **$34**. The risk-free interest rate is **12%** per annum with continuous compounding. Suppose ST is the stock price at the end of two months. What is the value of a derivative that pays off ST2 at this time?

Current price = $30

Up price = $34

Down price = $27

Risk-free rate = 12%

Option price = f

T = 2 months

For a riskless portfolio = -1 option

34 Δ – 6 = 27 Δ

Δ = 0.85

Present Value of portfolio today is

= (27 \* 0. 85) e ^ (-0.12 \* 2 / 12)

= 22.95 e ^ (-0.12 \* 2 / 12)

= 22.495

With current stock price, the value of Portfolio today is also

= 30 \* 0.85 – f

So, 22.495 = 30 \* 0.85 – f

f = 3.005

The derivative that pays off ST2

= **3.005**

3. (10 marks) Draw a diagram showing the variation of an investor’s profit and loss with the terminal stock price for a portfolio consisting of

(a) One share and a short position in one call option  
(b) Three shares and a short position in two call options   
(c) Two shares and a short position in three call options   
(d) Two shares and a short position in four call options

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4. (10 marks) A stock price is currently **$50**. It is known that at the end of **one** month it will be either **$52.5** or **$47.5**. The risk-free interest rate is **9%** per annum with continuous compounding. Use the no-arbitrage approach to find the value of a one-month European call option with a strike price of **$49**?

Current price = $50

Up price = $52.5

Down price = $47.5

Risk-free rate = 9%

Strike price = $49

Option price = f

T = 1 months

For a riskless portfolio,= -1 option

52.5 Δ – 4 = 47.5 Δ

Δ = 0.8

Present Value of portfolio today is

= (47.5 \* 0. 8) e ^ (-0.09 \* 1 / 12)

= 38 e ^ (-0.0075)

= 37.716

With current stock price, the value of Portfolio today is also

= 50 \* 0.8 – f

So, 37.716 = 50 \* 0.8 – f

f = 2.283

The value of a one-month European call option is

= **2.283**

5. (10 marks) A stock price is currently **$60**. It is known that over each of the next **two three-month periods** it will go up by **10%** or down **8%**. The risk-free interest rate is **11%** per annum with continuous compounding. Use the **risk-neutral** approach to find the value of a **six-month European** put option with a strike price of **$60**?

u = 0.1 (10%)

d = 0.08 (8%)

S = 60

T = 0.25 (3 months periods)

R = 0.11 (11%)

S \* e ^ (R \* T) = Su \* p + (Sd \* (1 - p))

60e^(0.11 \* 0.25) = (66 \* p) + (55.2 \* (1- p))

61.672 = 66p + 55.2 – 55.2p

6.472 = 10.8p

p = 0.5993

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**Put Option** = e ^ -2 (0.11 \* 0.25) \* ((0.5993 ^ 2 \* 0) + (2 \* (0.5993 \* (1 – 0.5993)) \* 1) + ((1 – 0.5993) ^ 2) \* 9.216)

= 0.9464 \* (0.48072 + 1.4797)

= **1.8553**

**6.** (10 marks) A bank loan department is trying to determine the correct rate for a 2-yr loan to be made two years from now. If current zero rates are 1-yr = 2%, 2-yr = 3%, 3-yr = 3.5%, 4-yr = 4.5%, the foward rate for the loan should be?

|  |  |
| --- | --- |
| Years | Zero rates |
| 1 | 2% |
| 2 | 3% |
| 3 | 3.5% |
| 4 | 4.5% |

Rf2 = (R2 \* T2 – R1 \* T1) / (T2 – T1)

= (0.03 \* 2 – 0.02 \* 1) / (2 - 1)

= 0.06 – 0.02 / 1

= 0.04

= **4%**

**7.** (10 marks) Suppose that the risk-free interest rate is **9%** per **annum** with continuous compounding and that the dividend yield on a stock index is **3%** per annum. The index is standing at **500**, and the futures price for a contract deliverable in **four** months is **520**. What arbitrage opportunities does this create?

Suppose FV = 500 \* e ^ ((0.09 – 0.03) \* 4 / 12)

= 500 \* e ^ 0.02

= **510.099**

Since Future price for a contract = **520**

The index’s future value is lower than contract’s future price (510.099 < 520)

The correct arbitrage should be

1. **Sell futures** contract
2. **Buy the stocks** underlying the index

**8.** (10 marks) The **6-month, 12-month, 18-month,** and **24-month** zero rates are **5%**, **5.5%**, **5.75%,** and **6%**, with semiannual compounding.

(a) What are the **rates** with continuous compounding?   
(b) What is the forward rate for the **6-month** period beginning in **18** months?   
(c) What is the value of an FRA that promises to pay you **6.5%** (compounded semiannually) on a principal of **$1** million for the **6-month** period starting in **18 months**?

(a) **Continuous compounding** = 2ln(1 + R / 2)

**6-month** = 2ln(1 + (0.05 / 2))

= 0.04938

= **4.938%**

**12-month** = 2ln(1 + (0.055 / 2))

= 0.05425

= **5.425%**

**18-month** = 2ln(1 + (0.0575 / 2))

= 0.05668

= **5.668%**

**24-month** = 2ln(1 + (0.06 / 2))

= 0.05911

= **5.911 %**

|  |  |
| --- | --- |
| Months | Zero rates |
| 6 | 5% |
| 12 | 5.5% |
| 18 | 5.75% |
| 24 | 6% |

(b)

Forward rate = (R24 \* T24 – R18 \* T18) / (T24 – T18)

= (0.05911 \* 2 – 0.05668 \* 1.5) / 2 – 1.5

= 0.0664

= **6.64% (Continuous compounding)**

For semi annual

0.0664 = 2ln(1 + R/2)

0.0332 = ln(1 + R/2)

e ^ 0.0332 = 1 + R/2

R = 0.0675

= **6.75% (Semiannual compounding)**

(c) Since Forward rate for 18-months is 6.75% but FRA only give 6.5%, we need to pay difference of interest

= 1,000,000 \* (0.065 – 0.0675) \* 6/12 \* (e ^ - 0.05911 \* 2)

= **-$1,110.6393**

**9.** (10 marks)A **futures contract** is used for hedging. Explain why the daily settlement of the contract can give rise to cash flow problems.

**Answer**

With Futures contract, so the buyer will need to pay a certain amount on a certain period in the future. So the price of the contract will depends on the movement of future price.

Since Future price is rely on marking to market price of the contract, so if the future’s rely asset price increases, the future price will also increase as well, then the owner will get margin call with a certain amount depends on current asset price that could lead to a rise of cash flow problems. Since there is a mismatch between cash inflows and out flow.

**10.** (10 marks) An investment bank can **borrow** or **lend** at LIBOR. Suppose that the **6-month** rate is **5%** and the **12-month** rate is **8%**. The rate that can be locked in for the period between **6** months and **12** months using an **FRA** is **7%**. What **arbitrage** opportunities are open to the bank? All rates are continuously compounded.

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**Answer**

Since we can **borrow** for **6 Months** at **5%** and enter **FRA** from **6 – 12 Months** at **7%**, so after that we can **lend** for **12 Months** at **8%** and earn a difference.

**11.** (10 marks) Trader A enters into a forward contract to buy gold for $1000 an ounce in one year. Trader B buys a call option to buy gold for $1000 an ounce in one year. The cost of the option is $100 an ounce. What is the difference between the positions of the traders? Show the profit per ounce as a function of the price of gold in one year for the two traders and plot the function.

**Answer**

Since Trader A enter a forward contract, so no matter what the price of gold in the future is, he will force to buy gold at the contract price even it will eventually lead to losses money if market price is lower than the contract price.

On the other hand, with Trader B, because he is holding an option, so he doesn’t need to exercise it unless the exercise price is lower than the spot price. If he decided not to exercise the contract, then his loss will be just **$100** following his cost of the option.

Gold price in one year = St

Delivery Price = K

For **Trader A**

Profit of Trader A = St - 1,000

Payoff for forward contract = St - K

For **Trader B**

Option’s price = 100

Profit of Trader A = max (St 1000, 0) – 100

= max (St 900, 0)

Payoff for forward contract = max (St – K, 0)

Chart

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**12.** (10 marks) Show that, if the futures price of a commodity is greater than the spot price during the delivery period, then there is an arbitrage opportunity. Does an arbitrage opportunity exist if the futures price is less than the spot price? Explain your answer.

**Answer**

If future price is less than spot price during the delivery period, an investor can take a long future contract and wait until delivery period and earn the difference later on.